

# Paper: VEXPA: Validated EXPonential Analysis through regular sub-sampling.pdf

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VEXPA: Validated EXPonential Analysis through regular sub-sampling

High noise experiment 5.2.

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- High noise experiment 5.2.

## Script environment

This script does depend on the random number generator state.

```
clear
close all
```

## High noise experiment 5.2.

For our second experiment we consider a signal  $\phi(t)$  defined by the parameters  $\beta_i, \gamma_i, \omega_i, \psi_i, i = 1, \dots, 12$  listed in Table 2. We perturb the samples with white circular Gaussian noise of increasing SNR. The total number of samples is again  $N = 300$ . The perturbed signal is then analysed using ESPRIT on the one hand and VEXPA (on top of ESPRIT) on the other. For the latter we choose  $u = 7$  and  $s = 6$ . We pass the correct model order  $n$  only to ESPRIT. The new VEXPA will detect it automatically.

Up to  $\text{SNR} = 20$  dB both ESPRIT and VEXPA closely approach the theoretical CRLB, as can be seen from Figure 10. In Figure 11 we show the retrieved  $\omega_i$ -values for both ESPRIT (top) and VEXPA (bottom). For higher noise levels (smaller SNR) the stand-alone ESPRIT method returns unreliable results, while the VEXPA method implemented on top of ESPRIT detects that the signal is heavily perturbed as none of the computed results is validated in the cluster analysis. Therefore VEXPA, in its standard implementation, does not return  $\lambda_i$  output.

```
params = params_from_table2;
signal = params.construct(300);
w = imag((log(params.b)*100))/(2*pi);
```

```

bsolver_esprit = BSolverEsprit('--nsamples',300,'--ncols',100,...
                                '--nrows', 201,'--nterms',12);

u = 7;
s = 6;
bsolver_vexpa = BSolverVexpa( '--bsolver', BSolverEsprit( ...
                                '--nterms',15) ...
                                , '--csolver', CSolverVandermondeLS ...
                                , '--rate' , u ...
                                , '--shift' , s ...
                                , '--M' , 8 ...
                                , '--u-epsilon', 0.1 ...
                                , '--u-minpts' , 6 ...
                                , '--s-epsilon', 0.1 ...
                                , '--s-minpts' , 5 ...
                                , '--plot' , false ...
                                , '--time' , false ...
                                );

SNRvec = 0:50;
N = numel(SNRvec);
nexpvec = [repmat(300,1,20),repmat(100,1,31)];
exp_total = sum(nexpvec);

X_esprit = [];
Y_esprit = [];
X_vexpa = [];
Y_vexpa = [];
sigma = zeros(1,N);
crlb_esprit = zeros(1,N);
crlb_vexpa = zeros(1,N);
timer = 0;
for j = 1:N
    SNR = SNRvec(j);
    fprintf('\nSNR = %d\n', SNR)
    nexp = nexpvec(j);
    sigma_tmp = zeros(1,nexp);
    omrt_esprit = zeros(12,nexp);
    omrt_vexpa = zeros(12,nexp);
    for k = 1:nexp
        tic
        flag = 50;
        if ~mod(k,flag)
            ETA = exp_total*(timer/flag);
            fprintf(' > Experiment %d/%d > ETA: %d sec\n',...
                    k,nexp,round(ETA))
        end
    end
end

```

```

        timer = 0;
    end
    signal.remove_noise;
    sigma_tmp(k) = signal.add_white_gaussian_noise(SNR,'db');

    b_esprit = bsolver_esprit.solve(signal);
    w_esprit = imag(log(b_esprit)*100)/(2*pi);
    [~,I] = min(abs(w(:)-w_esprit), [],2);
    omrt_esprit(:,k) = w_esprit(I);
    X_esprit = [X_esprit, repmat(SNR,1,numel(b_esprit))];
    Y_esprit = [Y_esprit,w_esprit];

    b_vexpa = bsolver_vexpa.solve(signal);
    w_vexpa = imag(log(b_vexpa)*100)/(2*pi);
    if isempty(w_vexpa)
        w_vexpa = 0;
    end
    [~,I] = min(abs(w(:)-w_vexpa), [],2);
    omrt_vexpa(:,k) = w_vexpa(I);
    X_vexpa = [X_vexpa, repmat(SNR,1,numel(w_vexpa))];
    Y_vexpa = [Y_vexpa,w_vexpa];

    exp_total = exp_total-1;
    timer = timer + toc;
end

sigma(j) = mean(sigma_tmp);

crlb_esprit(j)=sqrt(median(var(2*pi/100*omrt_esprit, [],2).^2));
crlb_vexpa(j)=sqrt(median(var(2*pi/100*omrt_vexpa, [],2).^2));
end

crlb_theory = zeros(1,N);
for k = 1:N
    [~,~,crlb_params,~] = params.crlb(sigma(k),300);
    crlb_theory(k) = sqrt(median(crlb_params.^2));
end
figure
semilogy(SNRvec,crlb_theory,'k-')
hold on
semilogy(SNRvec,crlb_esprit,'b-s')
semilogy(SNRvec,crlb_vexpa,'r-o')
title(['Figure 10. Variance of ESPRIT (blue) and VEXPA (red), ...
      ' compared to the Cramer-Rao lower bound.'])

figure;

```

```

plot(repmat(SNRvec(:),1,12),repmat(w,N,1),'k')
hold on
ylim([-50,50])
xlabel('SNR_{dB}')
ylabel('| \omega_i / 2\pi |')
title('Figure 11. (top, blue) Retrieved \omega_i by ESPRIT.')
scatter(X_esprit,Y_esprit,10,'filled','MarkerFaceAlpha',.2,...
'MarkerFaceColor','b')
title('Figure 11. (top, blue) Retrieved \omega_i by ESPRIT.')

```

```

figure;
plot(repmat(SNRvec(:),1,12),repmat(w,N,1),'k')
hold on
ylim([-50,50])
xlabel('SNR_{dB}')
ylabel('| \omega_i / 2\pi |')
title('Figure 11. (top, red) Retrieved \omega_i by VEXPA.')
scatter(X_vexpa,Y_vexpa,20,'filled','MarkerFaceAlpha',.2,...
'MarkerFaceColor','r')
title('Figure 11. (bottom, red) Retrieved \omega_i by VEXPA.')

```

```

SNR = 0
> Experiment 50/300 > ETA: 10657 sec
> Experiment 100/300 > ETA: 10795 sec
> Experiment 150/300 > ETA: 10724 sec
> Experiment 200/300 > ETA: 10616 sec
> Experiment 250/300 > ETA: 10684 sec
> Experiment 300/300 > ETA: 10547 sec

```

```

SNR = 1
> Experiment 50/300 > ETA: 10494 sec
> Experiment 100/300 > ETA: 10422 sec
> Experiment 150/300 > ETA: 10351 sec
> Experiment 200/300 > ETA: 10294 sec
> Experiment 250/300 > ETA: 10257 sec
> Experiment 300/300 > ETA: 10263 sec

```

```

SNR = 2
> Experiment 50/300 > ETA: 10146 sec
> Experiment 100/300 > ETA: 10110 sec
> Experiment 150/300 > ETA: 10074 sec
> Experiment 200/300 > ETA: 9985 sec
> Experiment 250/300 > ETA: 10060 sec
> Experiment 300/300 > ETA: 9840 sec

```

SNR = 3  
> Experiment 50/300 > ETA: 9849 sec  
> Experiment 100/300 > ETA: 9750 sec  
> Experiment 150/300 > ETA: 9714 sec  
> Experiment 200/300 > ETA: 9659 sec  
> Experiment 250/300 > ETA: 9570 sec  
> Experiment 300/300 > ETA: 9552 sec

SNR = 4  
> Experiment 50/300 > ETA: 9494 sec  
> Experiment 100/300 > ETA: 9420 sec  
> Experiment 150/300 > ETA: 9343 sec  
> Experiment 200/300 > ETA: 9280 sec  
> Experiment 250/300 > ETA: 9230 sec  
> Experiment 300/300 > ETA: 9345 sec

SNR = 5  
> Experiment 50/300 > ETA: 9124 sec  
> Experiment 100/300 > ETA: 9027 sec  
> Experiment 150/300 > ETA: 8968 sec  
> Experiment 200/300 > ETA: 8910 sec  
> Experiment 250/300 > ETA: 8870 sec  
> Experiment 300/300 > ETA: 8787 sec

SNR = 6  
> Experiment 50/300 > ETA: 8810 sec  
> Experiment 100/300 > ETA: 8897 sec  
> Experiment 150/300 > ETA: 8651 sec  
> Experiment 200/300 > ETA: 8546 sec  
> Experiment 250/300 > ETA: 8524 sec  
> Experiment 300/300 > ETA: 8440 sec

SNR = 7  
> Experiment 50/300 > ETA: 8398 sec  
> Experiment 100/300 > ETA: 8322 sec  
> Experiment 150/300 > ETA: 8277 sec  
> Experiment 200/300 > ETA: 8205 sec  
> Experiment 250/300 > ETA: 8143 sec  
> Experiment 300/300 > ETA: 8080 sec

SNR = 8  
> Experiment 50/300 > ETA: 8026 sec  
> Experiment 100/300 > ETA: 8070 sec  
> Experiment 150/300 > ETA: 7929 sec  
> Experiment 200/300 > ETA: 7896 sec  
> Experiment 250/300 > ETA: 7785 sec

> Experiment 300/300 > ETA: 7726 sec

SNR = 9

> Experiment 50/300 > ETA: 7688 sec  
> Experiment 100/300 > ETA: 7614 sec  
> Experiment 150/300 > ETA: 7560 sec  
> Experiment 200/300 > ETA: 7514 sec  
> Experiment 250/300 > ETA: 7437 sec  
> Experiment 300/300 > ETA: 7382 sec

SNR = 10

> Experiment 50/300 > ETA: 7334 sec  
> Experiment 100/300 > ETA: 7250 sec  
> Experiment 150/300 > ETA: 7194 sec  
> Experiment 200/300 > ETA: 7148 sec  
> Experiment 250/300 > ETA: 7081 sec  
> Experiment 300/300 > ETA: 7120 sec

SNR = 11

> Experiment 50/300 > ETA: 6946 sec  
> Experiment 100/300 > ETA: 6897 sec  
> Experiment 150/300 > ETA: 6822 sec  
> Experiment 200/300 > ETA: 6767 sec  
> Experiment 250/300 > ETA: 6731 sec  
> Experiment 300/300 > ETA: 6647 sec

SNR = 12

> Experiment 50/300 > ETA: 6598 sec  
> Experiment 100/300 > ETA: 6536 sec  
> Experiment 150/300 > ETA: 6477 sec  
> Experiment 200/300 > ETA: 6410 sec  
> Experiment 250/300 > ETA: 6343 sec  
> Experiment 300/300 > ETA: 6286 sec

SNR = 13

> Experiment 50/300 > ETA: 6233 sec  
> Experiment 100/300 > ETA: 6196 sec  
> Experiment 150/300 > ETA: 6240 sec  
> Experiment 200/300 > ETA: 6111 sec  
> Experiment 250/300 > ETA: 6047 sec  
> Experiment 300/300 > ETA: 5955 sec

SNR = 14

> Experiment 50/300 > ETA: 5896 sec  
> Experiment 100/300 > ETA: 5841 sec  
> Experiment 150/300 > ETA: 5803 sec

> Experiment 200/300 > ETA: 5727 sec  
> Experiment 250/300 > ETA: 5661 sec  
> Experiment 300/300 > ETA: 5607 sec

SNR = 15

> Experiment 50/300 > ETA: 5529 sec  
> Experiment 100/300 > ETA: 5471 sec  
> Experiment 150/300 > ETA: 5406 sec  
> Experiment 200/300 > ETA: 5348 sec  
> Experiment 250/300 > ETA: 5308 sec  
> Experiment 300/300 > ETA: 5219 sec

SNR = 16

> Experiment 50/300 > ETA: 5161 sec  
> Experiment 100/300 > ETA: 5238 sec  
> Experiment 150/300 > ETA: 5053 sec  
> Experiment 200/300 > ETA: 4987 sec  
> Experiment 250/300 > ETA: 4915 sec  
> Experiment 300/300 > ETA: 4858 sec

SNR = 17

> Experiment 50/300 > ETA: 4799 sec  
> Experiment 100/300 > ETA: 4747 sec  
> Experiment 150/300 > ETA: 4687 sec  
> Experiment 200/300 > ETA: 4617 sec  
> Experiment 250/300 > ETA: 4548 sec  
> Experiment 300/300 > ETA: 4495 sec

SNR = 18

> Experiment 50/300 > ETA: 4447 sec  
> Experiment 100/300 > ETA: 4381 sec  
> Experiment 150/300 > ETA: 4323 sec  
> Experiment 200/300 > ETA: 4263 sec  
> Experiment 250/300 > ETA: 4186 sec  
> Experiment 300/300 > ETA: 4117 sec

SNR = 19

> Experiment 50/300 > ETA: 4057 sec  
> Experiment 100/300 > ETA: 4014 sec  
> Experiment 150/300 > ETA: 3934 sec  
> Experiment 200/300 > ETA: 3869 sec  
> Experiment 250/300 > ETA: 3818 sec  
> Experiment 300/300 > ETA: 3762 sec

SNR = 20

> Experiment 50/100 > ETA: 3704 sec

```
> Experiment 100/100 > ETA: 3631 sec

SNR = 21
> Experiment 50/100 > ETA: 3576 sec
> Experiment 100/100 > ETA: 3522 sec

SNR = 22
> Experiment 50/100 > ETA: 3467 sec
> Experiment 100/100 > ETA: 3387 sec

SNR = 23
> Experiment 50/100 > ETA: 3330 sec
> Experiment 100/100 > ETA: 3268 sec

SNR = 24
> Experiment 50/100 > ETA: 3250 sec
> Experiment 100/100 > ETA: 3165 sec

SNR = 25
> Experiment 50/100 > ETA: 3186 sec
> Experiment 100/100 > ETA: 3025 sec

SNR = 26
> Experiment 50/100 > ETA: 2968 sec
> Experiment 100/100 > ETA: 2908 sec

SNR = 27
> Experiment 50/100 > ETA: 2849 sec
> Experiment 100/100 > ETA: 2784 sec

SNR = 28
> Experiment 50/100 > ETA: 2723 sec
> Experiment 100/100 > ETA: 2659 sec

SNR = 29
> Experiment 50/100 > ETA: 2612 sec
> Experiment 100/100 > ETA: 2554 sec

SNR = 30
> Experiment 50/100 > ETA: 2490 sec
> Experiment 100/100 > ETA: 2422 sec

SNR = 31
> Experiment 50/100 > ETA: 2364 sec
> Experiment 100/100 > ETA: 2301 sec
```

SNR = 32  
> Experiment 50/100 > ETA: 2244 sec  
> Experiment 100/100 > ETA: 2181 sec

SNR = 33  
> Experiment 50/100 > ETA: 2117 sec  
> Experiment 100/100 > ETA: 2062 sec

SNR = 34  
> Experiment 50/100 > ETA: 2003 sec  
> Experiment 100/100 > ETA: 1943 sec

SNR = 35  
> Experiment 50/100 > ETA: 1879 sec  
> Experiment 100/100 > ETA: 1818 sec

SNR = 36  
> Experiment 50/100 > ETA: 1756 sec  
> Experiment 100/100 > ETA: 1699 sec

SNR = 37  
> Experiment 50/100 > ETA: 1636 sec  
> Experiment 100/100 > ETA: 1575 sec

SNR = 38  
> Experiment 50/100 > ETA: 1524 sec  
> Experiment 100/100 > ETA: 1459 sec

SNR = 39  
> Experiment 50/100 > ETA: 1392 sec  
> Experiment 100/100 > ETA: 1337 sec

SNR = 40  
> Experiment 50/100 > ETA: 1274 sec  
> Experiment 100/100 > ETA: 1210 sec

SNR = 41  
> Experiment 50/100 > ETA: 1153 sec  
> Experiment 100/100 > ETA: 1091 sec

SNR = 42  
> Experiment 50/100 > ETA: 1030 sec  
> Experiment 100/100 > ETA: 969 sec

SNR = 43  
> Experiment 50/100 > ETA: 909 sec

```

> Experiment 100/100 > ETA: 849 sec

SNR = 44
> Experiment 50/100 > ETA: 793 sec
> Experiment 100/100 > ETA: 729 sec

SNR = 45
> Experiment 50/100 > ETA: 668 sec
> Experiment 100/100 > ETA: 606 sec

SNR = 46
> Experiment 50/100 > ETA: 545 sec
> Experiment 100/100 > ETA: 486 sec

SNR = 47
> Experiment 50/100 > ETA: 425 sec
> Experiment 100/100 > ETA: 365 sec

SNR = 48
> Experiment 50/100 > ETA: 304 sec
> Experiment 100/100 > ETA: 244 sec

SNR = 49
> Experiment 50/100 > ETA: 183 sec
> Experiment 100/100 > ETA: 122 sec

SNR = 50
> Experiment 50/100 > ETA: 62 sec
> Experiment 100/100 > ETA: 1 sec

```

Figure 10. Variance of ESPRIT (blue) and VEXPA (red), compared to the Cramer-Rao lower bound.

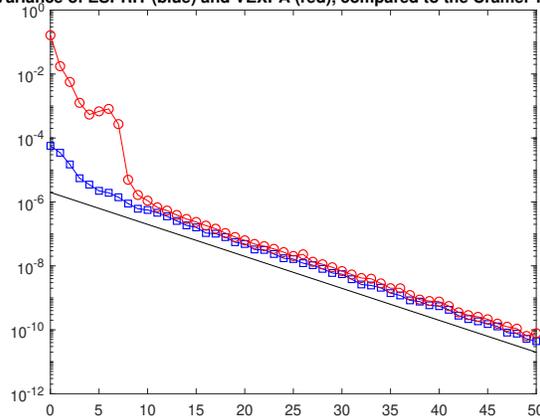


Figure 11. (top, blue) Retrieved  $\omega_i$  by ESPRIT.

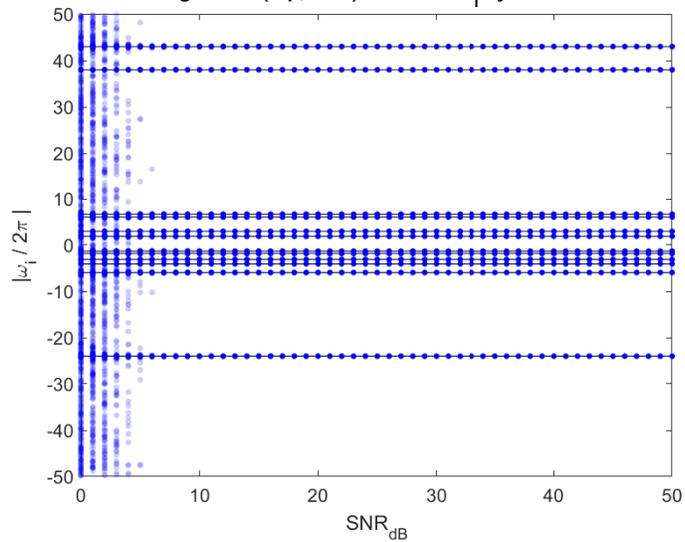


Figure 11. (bottom, red) Retrieved  $\omega_i$  by VEXPA.

